

**Athens 2014**  
2nd INTERNATIONAL CONFERENCE  
on Sustainable Solid Waste Management

# IMPACT OF LEACHATE FILTRATION ON SLOPE FAILURE POTENTIAL OF LANDFILL SIDE WALLS

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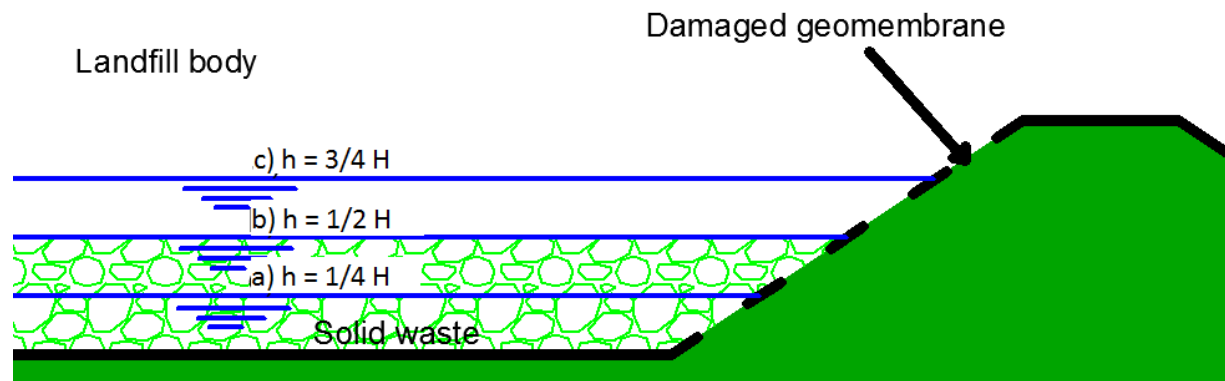
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# Content of the presentation

- Why are we analyzing this issue?
- Mathematical model / Governing equations
- Problem domain definition
- Numerical model
- Results
- Conclusions



# Why are we analyzing this issue?

- Organized solid waste disposal:

High income countries

Sanitary landfills

SW Incineration

Separation and recycling

...

Low and middle income countries

Sanitary landfills

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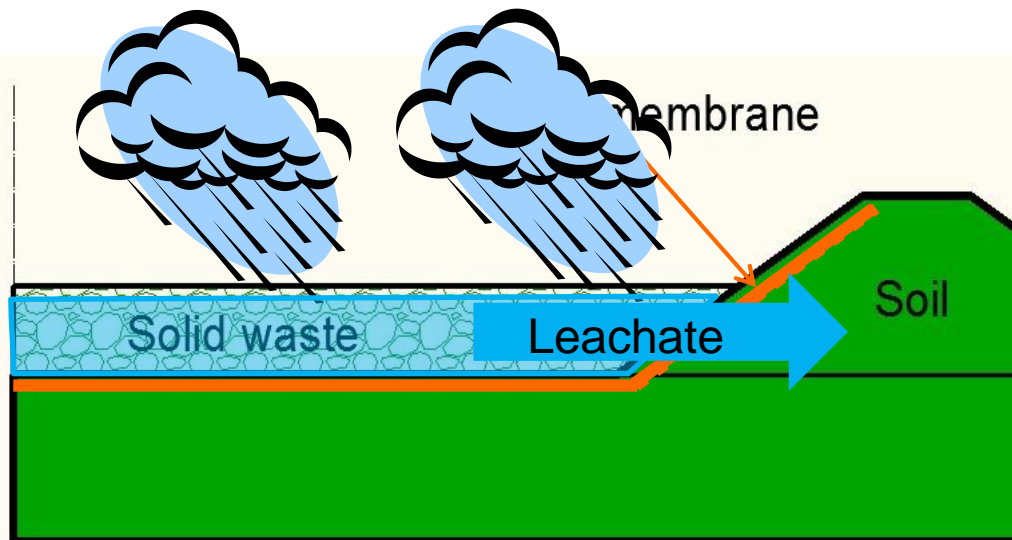
(around 84% of the global pop.

World Bank 2012)

- Different aspects of sanitary landfill design and exploitation have been examined and improved in past but...
- There are still potential hazards that require attention

# Why are we analyzing this issue?

- We are addressing the issue of leachate filtration through porous walls of landfills.
- Leachate is generated from the waste itself and by precipitation events (daily cover **is not** applied)



- Drainage system - out of order
- Heavy precipitation – (Serbia, Bosnia, Croatia 2014)
- Geomembrane – poorly built will deteriorate
- Gravity drives the leachate through porous wall
- **Seepage and buoyancy forces – shear slope failure?**

# Mathematical model / Governing equations

- Two issues:

1.) Steady state 2D filtration      2.) 2D poroelastic analysis

1.) Saturated / Unsaturated flow through porous media

## Darcy's law

$$q = \frac{Q}{A} = -K \frac{dh}{dl}$$

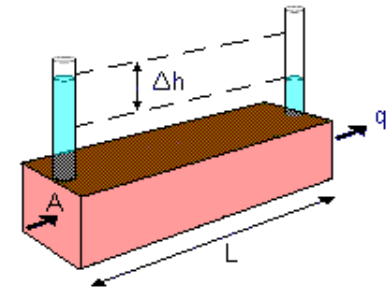
$h$  - fluid head/potential

$\frac{dh}{dl}$  - potential gradient

$K$  - hydraulic conductivity

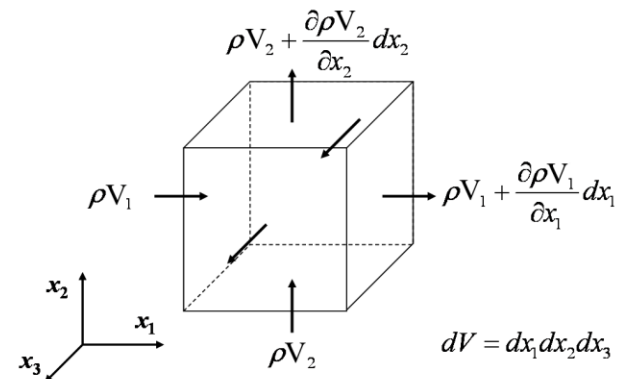
$Q$  - flow rate

$q$  - flow rate per unit area



## Laplace's equation

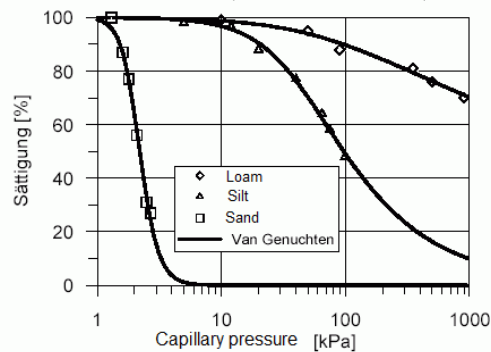
$$\frac{\partial}{\partial x} \left( -K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( -K_y \frac{\partial h}{\partial y} \right) = 0$$



# Mathematical model / Governing equations

## Soil water characteristic curve $\Psi - \theta_w$ (van Genuchten, 1980)

$$\theta = \frac{\theta_w - \theta_r}{\theta_s - \theta_r} = \left( \frac{1}{1 + (\alpha\Psi)^N} \right)^M$$



- $\theta$  - reduced saturation
- $\theta_w$  - water content for a particular soil
- $\theta_r$  - irreducible saturation
- $\theta_s$  - saturated water content
- $\Psi$  - capillary pressure
- $\alpha, N, M$  - model's parameters

coupled with relative permeability model (Mualem, 1976)

$$K_w(\theta_w) = k_{rw}(\theta_w)K$$

$$k_{rw} = \sqrt{\theta} \left( 1 - \left[ 1 - \theta^{1/M} \right]^M \right)^2$$

- $k_{rw}$  - relative permeability
- $K_w$  - unsaturated hydraulic conductivity

# Mathematical model / Governing equations

## 2.) 2D poroelastic analysis – Plane strain

### Hooke's law constitutive model

$$\varepsilon_{xx} = \frac{1}{E} \left[ (1-\nu^2) \sigma'_{xx} - \nu(1+\nu) \sigma'_{yy} \right]$$

$$\varepsilon_{yy} = \frac{1}{E} \left[ (1-\nu^2) \sigma'_{yy} - \nu(1+\nu) \sigma'_{xx} \right]$$

$$\varepsilon_{yx} = \frac{1+\nu}{E} \sigma'_{yx}$$

### Terzaghi's effective stress concept

$$\sigma'_{ij} = \sigma_{ij} + \alpha p \delta_{ij}$$

### Effective stress equilibrium

$$\frac{\partial \sigma'_{xx}}{\partial x} + \frac{\partial \sigma'_{yx}}{\partial y} = \alpha \rho_w g \frac{\partial h}{\partial x}$$

$$\frac{\partial \sigma'_{yy}}{\partial y} + \frac{\partial \sigma'_{yx}}{\partial x} = (\rho_t - \alpha \rho_w) g - \alpha \rho_w g \frac{\partial h}{\partial y}$$

Seepage body forces

Finally unknown displacements....

$$\frac{E}{2(1+\nu)} \nabla^2 u_x + \left[ \frac{\nu E}{(1-2\nu)(1+\nu)} + \frac{E}{2(1+\nu)} \right] \cdot \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial y \partial x} \right) = \rho_w g \frac{\partial h}{\partial x}$$

$$\frac{E}{2(1+\nu)} \nabla^2 u_y + \left[ \frac{\nu E}{(1-2\nu)(1+\nu)} + \frac{E}{2(1+\nu)} \right] \cdot \left( \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_x}{\partial x \partial y} \right) = (\rho_t - \rho_w) g + \rho_w g \frac{\partial h}{\partial y}$$

# Problem domain definition

## 1.) Steady state 2D filtration

$$\frac{\partial}{\partial x} \left( -K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( -K_y \frac{\partial h}{\partial y} \right) = 0$$

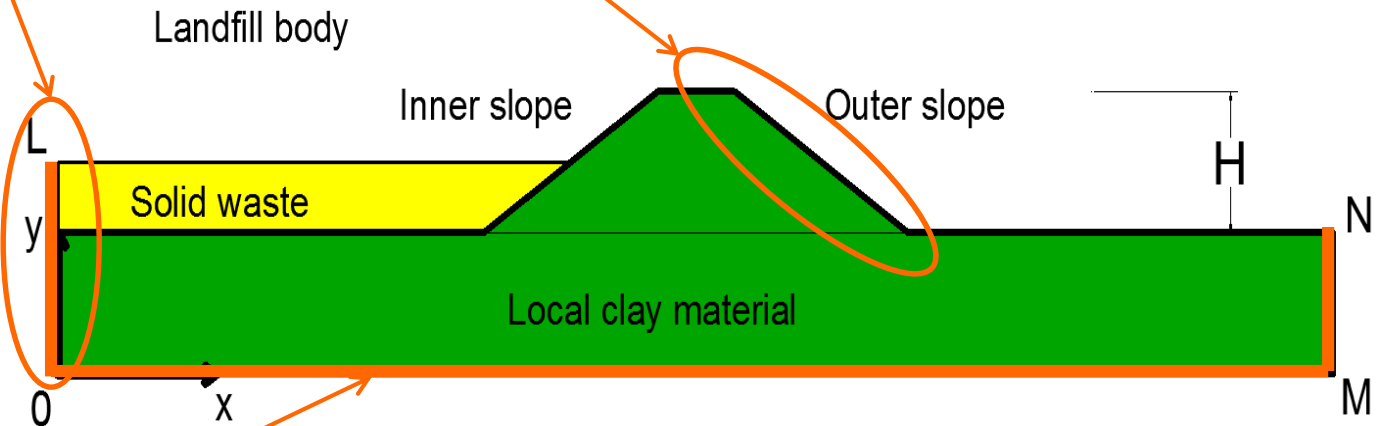
Boundary condition obtained through iterations

$$h(x_{RB}, 0) = 0$$

$$h(x_{RB}, y_{RB}) = y_{RB}$$

Boundary condition fixed

$$h(x_{LB}, y_{LB}) = h_L$$



no flow boundary

Another set is used for the poroelastic analysis



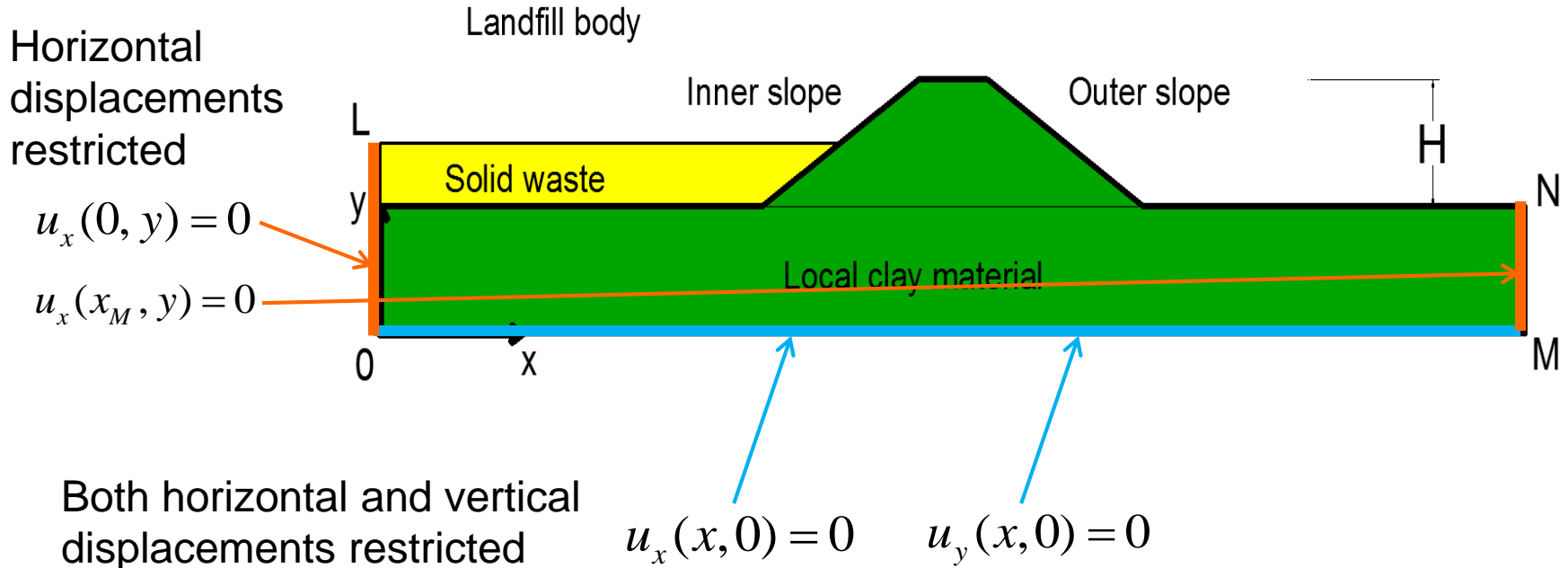


# Problem domain definition

## 1.) 2D poroelastic analysis – Plane strain

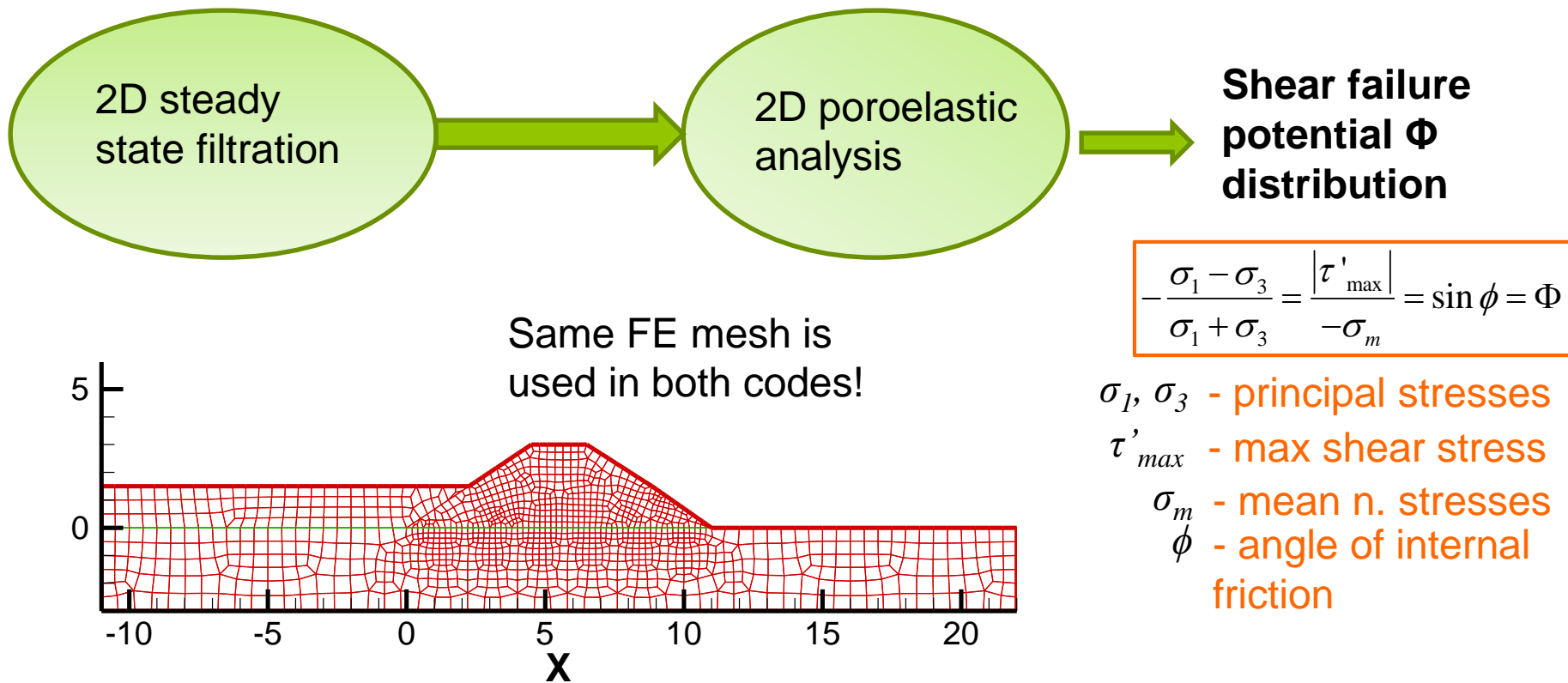
$$\frac{E}{2(1+\nu)} \nabla^2 u_x + \left[ \frac{\nu E}{(1-2\nu)(1+\nu)} + \frac{E}{2(1+\nu)} \right] \cdot \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial y \partial x} \right) = \rho_w g \frac{\partial h}{\partial x}$$

$$\frac{E}{2(1+\nu)} \nabla^2 u_y + \left[ \frac{\nu E}{(1-2\nu)(1+\nu)} + \frac{E}{2(1+\nu)} \right] \cdot \left( \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_x}{\partial x \partial y} \right) = (\rho_t - \rho_w) g + \rho_w g \frac{\partial h}{\partial y}$$



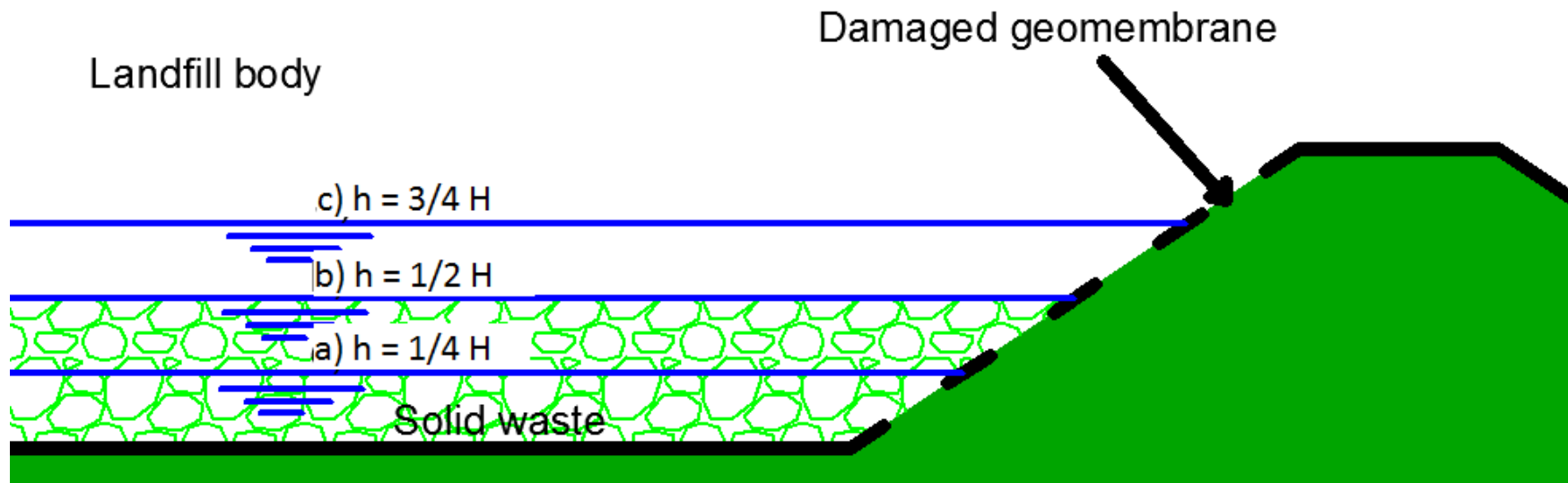
# Numerical model

- We employ two FE codes with quadrilateral elements:



# Results

- Nondimensionalization of the variables
- **Three scenarios** investigated and compared to **dry or normal operating conditions...**



- Outer slope toe of the wall is the critical area...

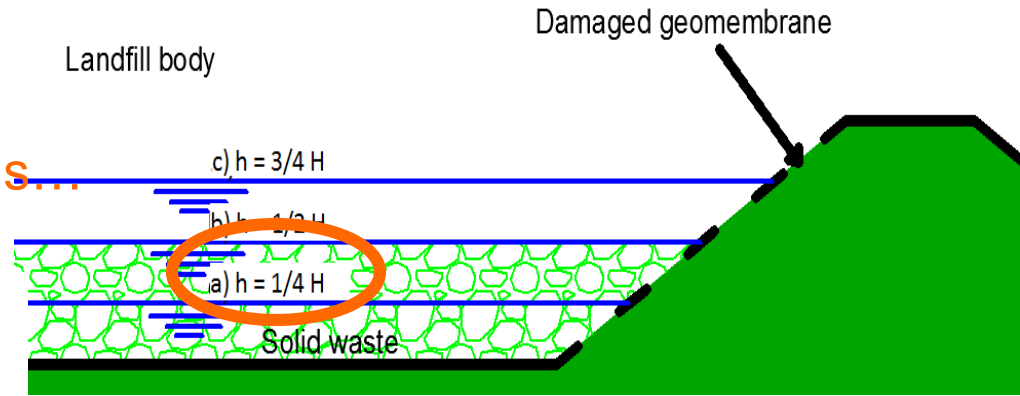
# Results

- Solid waste parameters for filtration analysis similar to gravel!

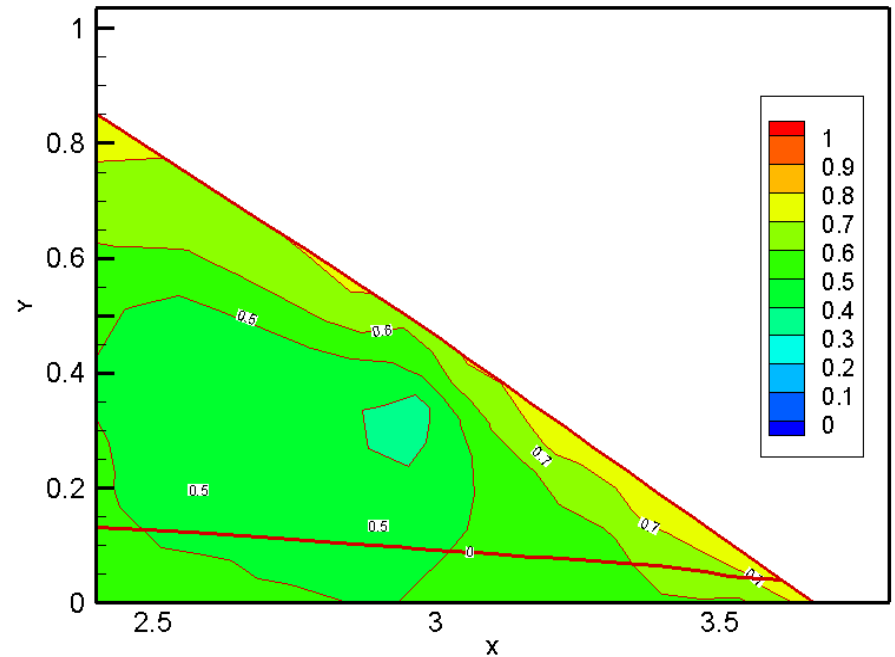
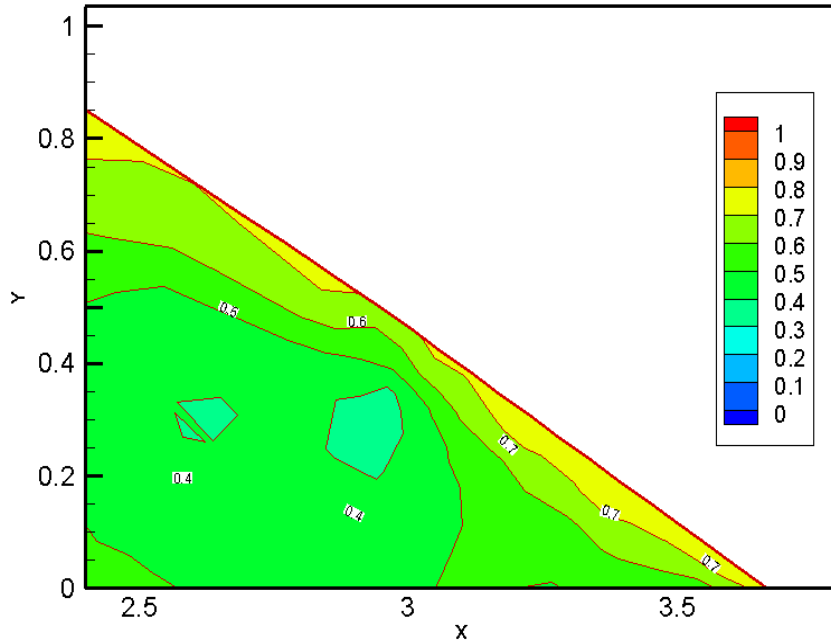
Parameters for elastic displacement analysis						
Material	Dry density	Saturated density	Poisson's ratio	Young's modulus		
	$\rho_{td}$	$\rho_{ts}$	$\nu$	E		
	[kg/m <sup>3</sup> ]	[kg/m <sup>3</sup> ]	[/]	[MPa]		
Local clay	1600	2000	0.3	50		
Solid waste	770	1000	0.33	1		
Parameters for groundwater flow analysis						
Material	Hydraulic conductivity x	Hydraulic conductivity y	Residual w. content	Saturated w. content	Model par.	Model par.
	$K_x$	$K_y$	$\theta_r$	$\theta_w$	$\alpha$	n
	[m/s]	[m/s]	[/]	[/]	[/]	[/]
Local clay	0.0000001	0.00000001	0.19	0.4	0.8	1.3
Solid waste*	0.0001	0.00001	0	0.23	2	2.9

# Results

Dry or normal operating conditions...

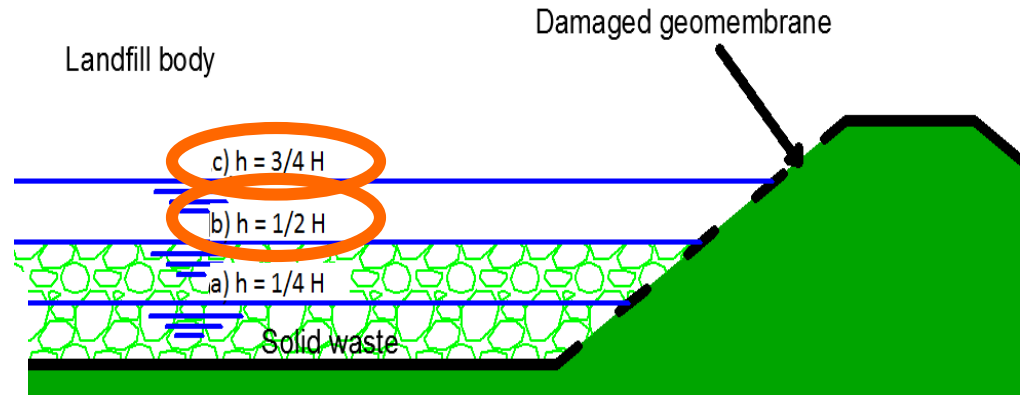
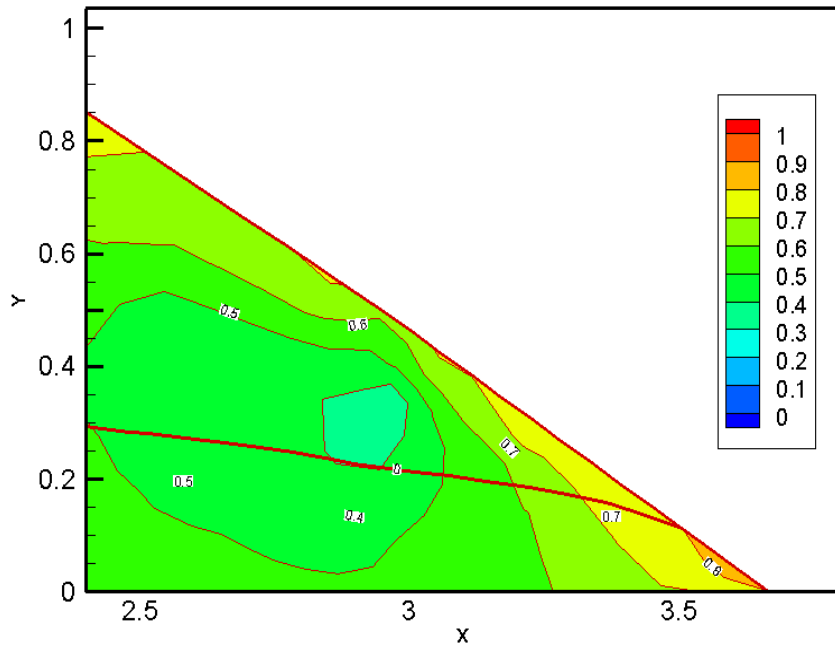


Scenario a)  $h = 1/4 H$ ...

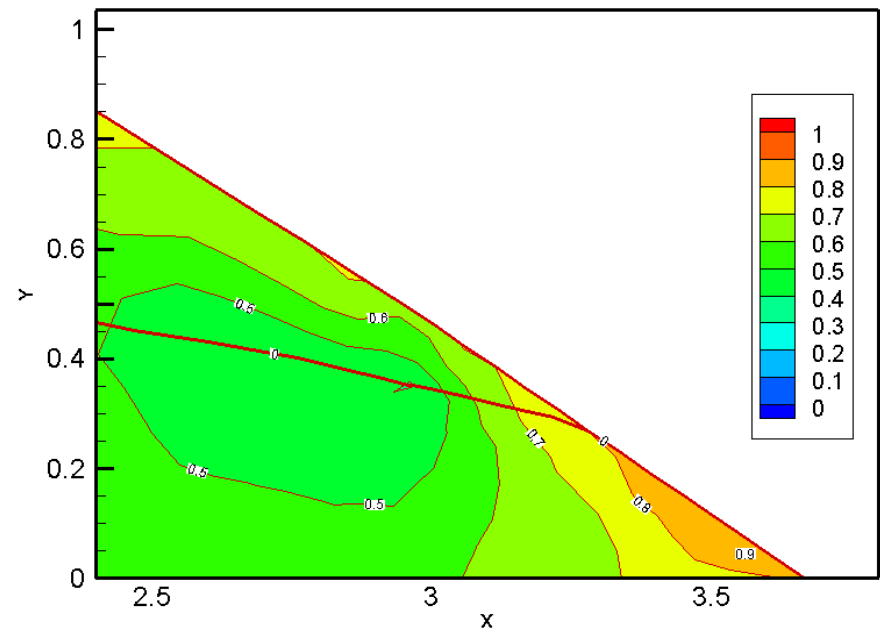


# Results

Scenario b)  $h = \frac{1}{2} H$ ...



Scenario c)  $h = \frac{3}{4} H$ ...



# Conclusions

- A numerical model for simulation of leachate flow in saturated and unsaturated porous media coupled with elastic displacement analysis was developed.
- We assumed that porous landfill walls are elastic and investigated the effects of leachate filtration for three scenarios  $h = \{1/4 H, 1/2 H, 3/4 H\}$  on the shear slope failure potential  $\Phi$
- Most significant effects were observed at the outer slope toe where the maximum increase of  $\Phi$  was about 30%.
- Since leachate usually contains a variety of the contaminants, our intention is to extend developed model by including contaminant transport analysis and investigate potential negative impacts on the environment.

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